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Generality of drought flow characteristics within the Arkansas River basin

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Abstract. Drought flow measurements from 11 river basins, contained within and surrounding the Cooperative Atmosphere-Surface Exchange Study (CASES) in the Walnut River watershed in Kansas, were subjected to a recession slope analysis. This analysis was based on the Dupuit-Boussinesq groundwater formulation and made use of the Historical Streamflow Daily Values published by the U.S. Geological Survey. The long- and short-term response features of the riparian aquifers were found to be practically identical to those observed in an earlier study in the Washita River basin in Oklahoma. As both study areas lie within the Osage section of the Central Lowland physiographic province, these findings support the applicability of regionalization and scaling of low-streamflow-related basin characteristics within the same geomorphic domain.

1. Introduction

The flow in natural river systems is typically generated by a variety of sources involving surface runoff and subsurface flow from riparian aquifers. The subsurface contributions to the natural flow consist of two principal types in nonregulated river systems. Base or drought flow is the groundwater outflow during periods of no precipitation; the other type of subsurface contribution consists of the outflows from more shallow soil layers as a direct result of storm events.

This paper presents an analysis of drought flows measured by the U.S. Geological Survey (USGS) at and around the Cooperative Atmosphere-Surface Exchange Study (CASES) experimental area in the Walnut River watershed in Kansas; it utilizes a technique developed by *Brutsaert and Nieber* [1977] (hereinafter referred to as BN77) to obtain the relevant basin-scale aquifer parameters. The results of this analysis are compared to those obtained in the Washita River basin in Oklahoma by *Brutsaert and Lopez* [1998] (hereinafter referred to as BL98) with an independent data set of measurements from the U.S. Department of Agriculture-Agricultural Research Service (USDA-ARS). The main objective is to test the similarity and statistical homogeneity of drought flow characteristics within the same geomorphic region. The characterization of the hydraulic behavior of riparian aquifers on this basis should allow the improved prediction of drought flows at ungaged stream locations for the entire region of interest.

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2. Study Area

To regionalize the results of the Washita River basin, a total area of 140,000 km^2 was established as the geomorphic delineation surrounding the CASES and Washita River basin sites. This was done to provide similar topographic and physiographic features such as relief, type of land use, soils, and geologic history. The delineated study area constitutes the Osage section of the physiographic province of the Central Lowland [*Hunt,* 1967]. In popular usage the study area, which occupies parts of Kansas and Oklahoma, is often referred to as part of the Great Plains, but technically, it should be specified as the Central Lowland [*Hunt,* 1967; *Thornbury,* 1965; *Loomis,* 1937]. The approximate latitude (north) and longitude (west) coordinates of the four corners of the delineated study area are 39°and 95°, 39°and 98°, 35°and 96°, and 35°and 100°, respectively.

At the time of this study the area contained a total of 11 USGS gaging stations that provided daily drought flow data unaffected by major upstream impoundments, diversions, or other modifications (see Table 1). The reported upstream total stream lengths, L, were obtained from USGS 7.5 min series topographic maps. The lengths of the intermittent flow (blue stipple lines) and the perennial flow (blue solid lines) channels were measured using a planimeter. Similar to the result in BL98, a strong relationship was found between drainage area A (in km^2) and stream length L (in kilometers) by ordinary least squares of the logarithms, namely,

$$A = 0.451 L^{1.062} \tag{1}$$

with a correlation coefficient r=0.996.

Table 1. Basin Characteristics

Name	Station	POR, years	A, km^2	L, km	a_1	a_3
Cedar Creek near Cedar Point,KS	07180500	1964 to 1973	285	444	0.0333	0.0125
Cole Creek near Degraff, KS	07146570	1961 to 1980	78	143	0.0308	0.524
Marais Des Cygnes River near Reading, KS	06910800	1969 to 1993	458	*697	0.0333	0.00287
Otter Creek at Climax, KS	07167500	1946 to 1993	334	504	0.0333	0.00536
Slate Creek at Wellington, KS	07145700	1969 to 1993	399	549	0.0273	0.0249
Verdigris River near Madison, KS	07165700	1955 to 1976	469	*774	0.0315	0.00175
Council Creek near Stillwater, OK	07163000	1934 to 1993	80	140	0.0294	0.274
Dry Creek near Kendrick, OK	07243000	1955 to 1993	179	265	0.0306	0.125
Flat Rock Creek, St Cincinnati Avenue at Tulsa, OK	07177650	1987 to 1996	21	34	0.0441	2.244
Salt Creek near Okeene, OK	07158400	1961 to 1979	508	*644	0.0333	0.00873
Snake Creek near Bixby, OK	07165550	1961 to 1970	130	214	0.0333	1.060
Arithmetic mean			267	287	0.0327	0.38937
Geometric mean			191	216	0.0325	0.04801
Standard deviation			178	190	0.0043	0.69696

* Refers to large reservoir(s)(greater than 0.04 km^2) located within the drainage area.

POR, period of record; A, drainage area size; L, upstream stream length; KS, Kansas; OK, Oklahoma. Value a_1 is in days^{-1} and a_3 is in (m^3/s)$^{-2}$day^{-1}

3. Theoretical Model

The time when a recession flow begins in a stream hydrograph $Q=Q(t)$ is difficult to establish. The technique of BN77 eliminates this problem by examining the time derivative of the flow rate,

$$\frac{dQ}{dt} = \phi(Q) \tag{2}$$

where $\phi(\ )$ is a function characteristic for a given catchment. The derivative dQ/dt can be estimated using a finite difference form as follows:

$$\frac{(Q_{i+1} - Q_i)}{\Delta t} = \phi\left(\frac{Q_{i+1} + Q_i}{2}\right) \tag{3}$$

Here Q_i is the flow rate at time t, and Q_{i+1} is the flow rate at time $t+\Delta t$.

For several available solutions to the *Boussinesq* [1903] equation for the ideal case of outflow from an unconfined rectangular aquifer discharging into a fully penetrating stream, in BN77 it was shown that (2) can be expressed as a power relationship

$$\frac{dQ}{dt} = -aQ^b \tag{4}$$

where a and b are constants. These constants can be used to describe the hydrograph recession.

The first solution to the *Boussinesq* [1903] equation of interest here involves linearization of the nonlinear partial differential equation and subsequent solution using a Fourier expansion to describe the outflow rate. When considering a long period of drainage, without rainfall input, only the first term of this series has to be retained, while the higher-order terms become negligible. This solution results in the following constants:

$$a_1 = \frac{\pi^2 kpDL^2}{fA^2} \tag{5a}$$

$$b_1 = 1 \tag{5b}$$

where k is the hydraulic conductivity, f is the drainable porosity, D is the aquifer thickness, L is the upstream stream length, A is the watershed area, and where p, which is approximately 0.3465 (see BN77), is introduced to account for the approximation as a result of the linearization. There is a second exact solution of the Boussinesq equation, which produces $b=3/2$ in (4) and which can be used to describe outflows for large values of time. However, as is discussed in section 4.1, preliminary tests in the manner of BL98 revealed that the long-time behavior was best described by $b=1$, as given by (5). Therefore this solution, with $b=3/2$, need not be presented here.

A third exact solution of interest, which was developed by *Polubarinova-Kochina* [1962], applies Boltzmann's transformation to *Boussinesq's* [1903] equation. This approach requires the assumption that the aquifer boundaries at the divide have no effect on the outflow, so solution is valid only for small periods of time after the inception of drought flow conditions. With this third solution the outflow rate evolves as $t^{-1/2}$, and the constants of (4) are as follows

$$a_3 = \frac{1.1334}{kfD^3L^2} \tag{6a}$$

$$b_3 = 3 \tag{6b}$$

Whenever measured values of dQ/dt are plotted versus Q on logarithmic scales, the data appear as a cloud. As described in BN77 and BL98, the baseflow characteristics of the watershed can be deduced from the lower straight line envelopes of this cloud. Indeed, the rate of decrease of groundwater flow into a natural channel can generally be assumed to be smaller than other sources such as overland flow and channel storage. Therefore for any given value of Q, the drought flow behavior can be associated with the smallest $|dQ/dt|$ value in the available data, i.e., the lower envelope. Conversely, however, for any given $|dQ/dt|$ these lower envelopes also represent the largest observed value in the available recession data. Thus they provide the relationship (2), for major runoff events, when the entire upstream catchment has been subjected to rainfall input, and is then contributing to the flow at the outlet.

4. Methodology

4.1. General Features of Aquifer Behavior

To validate the usage of the long-time solution (5) to the *Boussinesq* [1903], equation a methodology was adopted similar to that of BL98. Thus the organic correlation (OC) was applied to the cloud of all available data points plotted according to (3) for all 11 gaging stations; this regression technique was preferred because it accounts for the indeterminacy of which of the two variables considered is the dependent one [*Hirsch and Gilroy*, 1982]. The geometric mean slope generated this way was 1.163 with a standard deviation of 0.152. This result is close to unity as required in (5b) for the mean long-time recession behavior. It indicates that in spite of the scatter, the watersheds in the study area behave on average as linear storage elements similar to those in the Washita River basin; the geometric mean value obtained in the earlier study was 0.94 with a standard deviation of 0.14. This result is also consistent with the findings of *Vogel and Kroll* [1992] in Massachusetts.

4.2. "Step" Shape

A characteristic that appears on the majority of the plots of $-dQ/dt$ versus Q is a "step" shape between log cycles at the long-time recession flow envelope (see Figure 1). This step shape typically begins at flow rates 1 to 2 orders of magnitude smaller than the largest flow rates recorded at a station. This well-known feature is attributable to the resolution and rounding of the daily flow data provided by the USGS, since they are reported to at most three significant figures.

This lower envelope was clearly defined on the plots of $-dQ/dt$ versus Q in BL98, which did not suffer from this step problem; the reason for this was that the data of the Washita River basin had not been subjected to the same kind of rounding as the USGS data. One method proposed by *Troch et al.* [1993] to account for the uncertainty and scatter of data near the envelope is to exclude 5 to 10% of the data outside the envelope. However, this method could not be implemented to the plots generated in this study, because of this step shape. Therefore it was decided to place two envelopes passing through the upper and lower corners of the step shape boundary and to take the geometric mean line between them. This procedure was applied to all 11 basins and an example is shown in Figure 1. This way, consistent long-time recession envelopes were generated on all plots, with a_1 values similar to those of BL98.

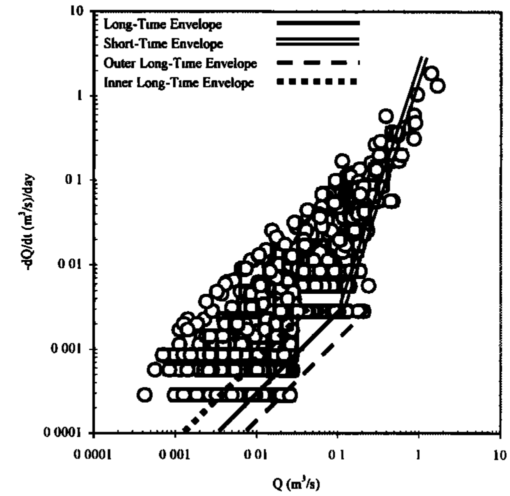

Figure 1. Graphical procedure to estimate long-time and short-time recession envelopes for Council Creek near Stillwater, Oklahoma (station 07163000).

4.3. Estimation of a_1 and a_3

For each of the 11 basins the values of a_1 and a_3 can be determined by examining the two lower envelopes with slopes of 1 and 3 on the cloud of points from the $-dQ/dt$ versus Q plots. The procedure is illustrated in Figure 1 and the values of a_1 and a_3 are listed in Table 1 for all 11 basins. The geometric mean value of a_1 was 0.0304 day^{-1} with a standard deviation of 0.0033. The geometric mean value for a_3 was 0.0480 (m^3/s)$^{-2}$day^{-1} with a standard deviation of 0.697. The corresponding mean results generated in BL98 were 0.0283 and 2.592, respectively; while the a_1 values are quite comparable, the a_3 values are not; this issue is further discussed below in section 5.4.2.

5. Results

5.1. Long-Term Behavior

The characteristic timescale for drought flow drainage within a basin can be represented by examining ${a_1}^{-1}$, as described in BL98. The geometric mean value of ${a_1}^{-1}$ is of the order of 32.89 days with a standard deviation of 2.89 days. This result is almost identical to the result of 35.4 days of BL98 for the Washita River basin.

Another important parameter in unconfined groundwater flow is the hydraulic diffusivity; it arises naturally in the Boussinesq equation as

$$D_h = \frac{kpD}{f} \tag{7}$$

and can be written in terms of a_1 (see BL98) as

$$D_h = a_1 \pi^{-2} \left(\frac{A}{L} \right)^2 \tag{8}$$

As illustrated in Figure 2, the values of D_h calculated with (8) in this study appear to be described by a lognormal distribution similar to those of BL98. This was confirmed by a Lilliefors test [*Wilks*, 1995], which indicated that the null hypothesis of the D_h values in this study being drawn from a lognormal distribution was not rejected even at the 20% level. The geometric mean value was 0.0147 m^2/s with a standard deviation of 0.00323 m^2/s, whereas in BL98 the geometric mean was 0.0205 m^2/s.

A difference of mean under independence test [*Wilks*, 1995, p. 122] was conducted both on the arithmetic means and on the log-transformed mean values of D_h obtained in this study and those of BL98. It was found at the 1% level that the D_h values obtained herein and in BL98 are probably drawn from the same population. However, the test did not confirm this at the higher percent levels.

5.2. Short-Term Behavior

The hydraulic desorptivity, which is defined by (see BL98)

$$D_{eh} = 0.6642(kf)^{1/2} D^{3/2} \tag{9}$$

and can be related to the short-term response constant a_3 by

$$D_{eh} = (2a_3 L^2)^{-1/2} \tag{10}$$

The Lilliefors test [*Wilks*, 1995] even at the 20% level strongly indicated that the desorptivities calculated from the a_3 values in Table 1 can be described by a lognormal distribution. A plot of these values in this study is presented in Figure 2. The geometric mean value is 0.00318 $m^2s^{-1/2}$ with a standard deviation of 0.001734 $m^2s^{-1/2}$; again, this is close to 0.00353 $m^2s^{-1/2}$ obtained in BL98.

To determine how similar the values of D_{eh} were in this study to the ones in BL98, again the difference of mean under independence test [*Wilks*, 1995] was conducted both on the arithmetic means and on the means of the logarithms. Even at the 20% level, in both cases it was found that the values of D_{eh} in this study were likely drawn from the same population as those in BL98.

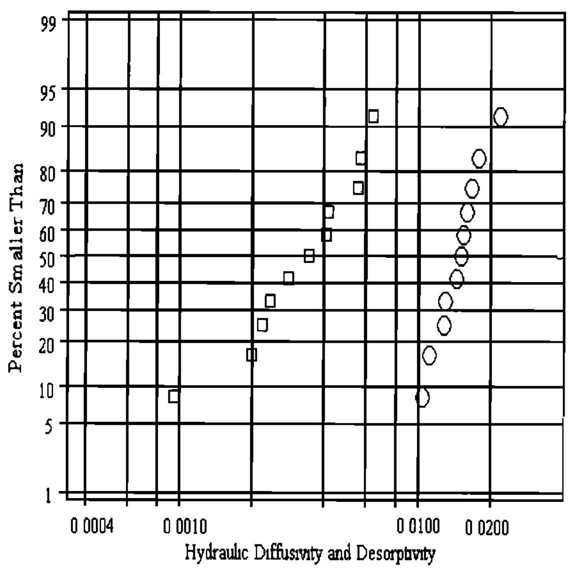

Figure 2. Distribution of the hydraulic diffusivity D_h (m^2/s) (ellipses) and the hydraulic desorptivity D_{eh} ($m^2s^{-1/2}$) (squares) shown with lognormal coordinate axes; "percent smaller than" was calculated by means of the Weibull plotting position $m/(n+1) = m/12$, in which m is the rank of the item.

5.3. Aquifer Parameters k, f, and D

Equations (5a) and (6a) contain three unknowns between them. Therefore in the absence of a third equation, one of the three must be estimated somehow, before the remaining two can be calculated. This was done in two ways. The first approach used an estimate for f which was then used in a combination of (5a) and (6a) to obtain an expression for the aquifer thickness D in terms of f. The resulting equation after this combination is the following:.

$$D = \left[\frac{1.1334p}{a_3 a_1} \right]^{1/2} \left(\frac{\pi}{fA} \right) \tag{11}$$

When the aquifer thickness was calculated, the hydraulic conductivity k could be calculated from (5a). For this first method the required drainable porosity f was assumed to have a value of 0.017 between those from BL98 and *Troch et al.* [1993], and this was then used to calculate the remaining aquifer parameters k and D for each basin. The distributions of k and D are presented in Figures 3 and 4, respectively. The geometric mean value of k is 0.00186 m/s with a standard deviation of 0.00177 m/s. The geometric mean value of D is 1.162 m with a standard deviation of 0.643 m.

The second approach assumed the a priori knowledge of the aquifer thickness to calculate the drainable porosity and the hydraulic conductivity. By combination of (5a) and (6a) the resulting expression for the hydraulic conductivity k in terms of the aquifer thickness D is the following:

$$k = \left[\frac{1.1334a_1}{\pi^2 p a_3} \right]^{-1/2} \frac{A}{(DL)^2} \tag{12}$$

When the hydraulic conductivity was calculated, the drainable porosity f could be calculated from (6a). The aquifer thickness selected was 1.5 m, a value similar to those of BL98. The distributions of the resulting

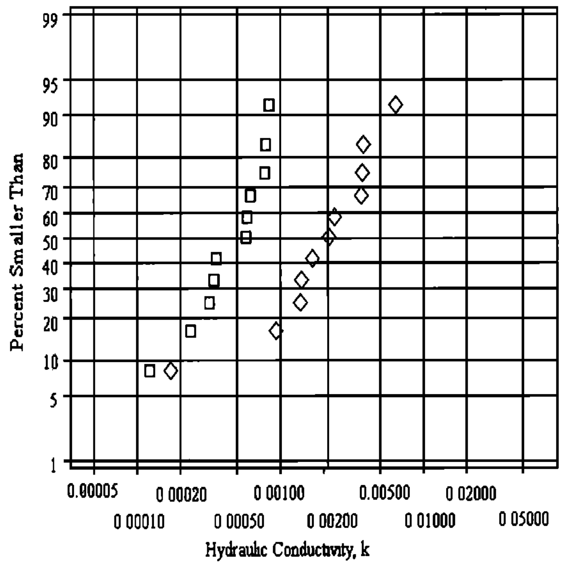

Figure 3. Distribution of the hydraulic conductivity k (m/s) for method 1 (diamonds) and method 2 (squares) shown with lognormal coordinate axes; "percent smaller than" was calculated by means of the Weibull plotting position $m/(n+1) = m/12$, in which m is the rank of the item.

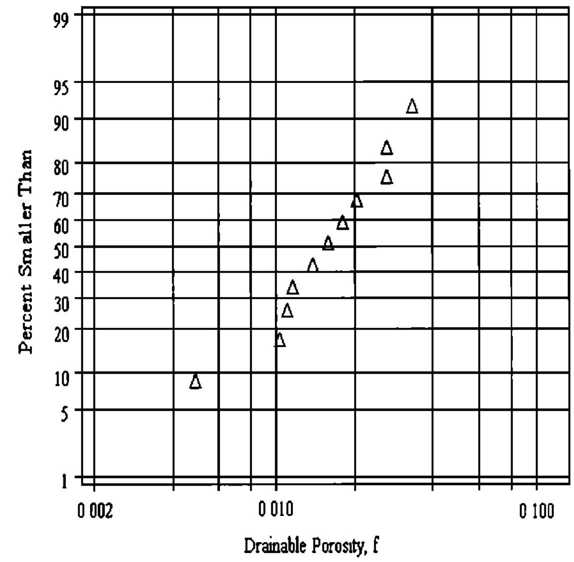

Figure 5. Distribution of the drainable porosity f for method 2 shown with lognormal coordinate axes; "percent smaller than" was calculated by means of the Weibull plotting position $m/(n+1) = m/12$, in which m is the rank of the item.

k and f are presented in Figures 3 and 5, respectively. Again, these distributions appear similar to the plots generated in BL98. Similarly, the Lilliefors test on these data suggest that k is better described by the lognormal distribution than by the normal distribution, whereas f can be described equally well by the lognormal and by the normal distribution. The geometric mean value of k is 0.000439 m/s with a standard deviation of 0.000244 m/s. The geometric mean value of f is 0.0155 with a standard deviation of 0.00859.

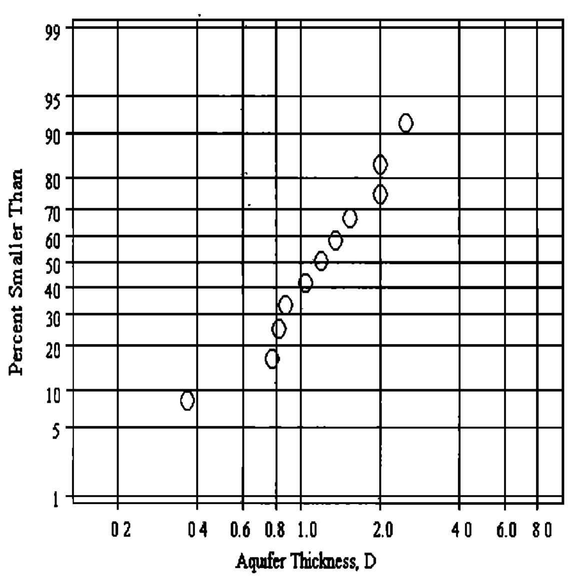

Figure 4. Distribution of the aquifer depth D (m) for method 1 shown with lognormal coordinate axes; "percent smaller than" was calculated by means of the Weibull plotting position $m/(n+1) = m/12$, in which m is the rank of the item.

Although method 2 appeared to give more reasonable results, the aquifer parameters calculated by both methods are similar to those of BL98. For the Washita basin the mean values obtained were D=1.31 m (directly measured), k=0.000757 m/s, and f=0.0167.

5.4. Scale Dependency of Hydraulic Parameters

5.4.1. Long-term hydraulic parameter. A simple correlation analysis was adopted from BL98 to check the scale dependence of the hydraulic parameters in that paper. This technique compared ${a_1}^{-1}$ to the basin scale which was represented equally well by either L or A due to the strong correlation reported in (1). With the present data, given in Table 1, linear regression of the logs produces the following power relationship:

$$ {a_1}^{-1} = (18.417)L^{0.102} \quad (13) $$

where L is in kilometers and ${a_1}^{-1}$ is in days (with r=0.726). Equation (5a) reports that a_1 is proportional to $(L/A)^2$; combining (5a) with (13), one obtains that A is proportional to $L^{1.051}$. This is very close to the power 1.062 reported in (1), which indicates that k, f, and D can be assumed to be scale independent. Actually, it is the combination quantity $(kpD/f) = D_h$ in (5a) which is shown here to be independent of the scale of the basin. This is consistent with the findings for the Washita River basin.

5.4.2. Short-term hydraulic parameter. In the graphical determination of a_3, it was noted that slight shifts in the placement of the short-time recession flow envelopes generated widely varying values of a_3. If it can be assumed, as indicated in the previous section, that k, f, D, and therefore also the quantity (kfD^3) can be taken as independent of the scale of the basin, (6a) suggests that the hydraulic parameter a_3 is close to inversely proportional to the upstream stream length squared. Accordingly, by imposing a slope of -2 on a log-log plot, the following was derived graphically with the data in Table 1:

$$a_3 = (4 \times 10^3)L^{-2} \tag{14}$$

where L is in kilometers and a_3 is in $((\mathrm{m}^3/\mathrm{s})^{-2}\mathrm{day}^{-1})$. Actually, calculation of the linear regression with the same data yields the following:

$$a_3 = (1.7 \times 10^5)L^{-2.6} \tag{15}$$

(with r=-0.905) in which for the data under consideration the power 2.6 is not all that different from 2. In any event, to test the assumption on which (14) is based, a_3 values were calculated with it for all 11 basins, and these were then used to place the short-term envelopes on the graphs. The difference between the graphically placed envelopes and those predicted by (14) was very small; a typical example of this is shown in Figure 6. This indicates that a_3 is an insensitive variable to moderate changes in the power of L. Put differently, it confirms that (6a) is valid and that the quantity (kfD^3) or D_{eh} is indeed fairly scale independent. This same conclusion was reached in BL98, further reinforcing the present findings. This also explains the earlier observation in section 4.2 why the mean value of a_3 observed in the Washita River basin is so different from that obtained in the present study. The reason is that the mean stream length in BL98 was around 37 kilometers, whereas herein, it is 216 kilometers.

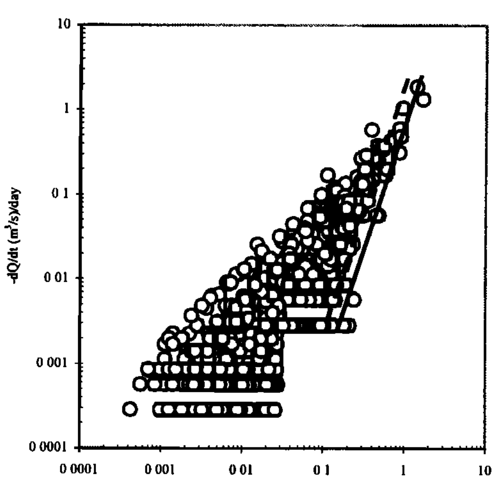

Figure 6. Plot of -dQ/dt versus Q for Council Creek near Stillwater, Oklahoma (station 07163000), comparing the short-term envelopes (see equation (2) with (b=3)), as placed graphically (dashed line) and as obtained with equation (14) (solid line).

6. Conclusions

Both short- and long-term drought flow characteristics of 11 river basins were found to be consistent with those predicted by Dupuit-Boussinesq hydraulic theory. This theory describes unconfined groundwater outflow from riparian aquifers with constant D, k, and f.

The long-term drainage behavior of the aquifers suggests that their response characteristics are close to linear, with typical exponential decay functions of time; their short-term behavior was shown to be consistent with Boltzmann similarity, with its typical $t^{-1/2}$ evolution.

The resulting values of the recession parameters a_1 and a_3, the hydraulic diffusivity $D_h = 0.35kD/f$, the hydraulic desorptivity $D_{eh} = 0.66(kf/D^3)^{1/2}$, the hydraulic conductivity k, the drainable porosity f, the aquifer depth D, and their respective scale dependence, obtained in the present study with USGS measurements, were in remarkable agreement with those obtained in an independent study of the Washita River basin with USDA-ARS measurements. These findings support the hitherto untested notion that within the same geomorphic region (in this case the Osage section of the Central Lowland province), drought flow characteristics and riparian aquifers exhibit a high degree of similarity and statistical homogeneity; this can serve as the basis for their regionalization and possibly the prediction of low-flow regimes at ungaged river sites.

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